Lecture 36

Monday, April 4, 2022 8:36 AM

* Prayer
* Spiritual thought: President Nelson was the first Church leader whose talk I attended. That was in 2014. He is a man of wisdom.

Line integral: $\int_C f\,ds$, $\int_C F \cdot dr$

infintesimal length

infinitesimal vector

$$ds = |dr|$$
$$= |(dx, dy, dz)|$$
$$= \sqrt{x'^2 + y'^2 + z'^2}\,dt$$

The length of a wire frame is

$$\int_C ds \quad (\text{mass density} = 1) \quad = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\,dt$$

How about area of a surface?

A surface can be the graph of a function $f(x,y)$, or not. We need to parametrize a surface.

Curve: $r(t) = (x(t), y(t), z(t))$ $\quad a \le t \le b$

Each t identifies for us a point on the curve.

Surface: $r(u,v) = (x(u,v), y(u,v), z(u,v))$, $(u,v) \in R$

Each pair $(u,v) \in R$ identifies for us a point on the surface.

Ex: elliptic paraboloid $z = x^2+y^2$.

This is the graph of function $f(x,y) = x^2+y^2$.

Parametrization: $\begin{cases} x = u \\ y = v \\ z = u^2+v^2 \end{cases}$

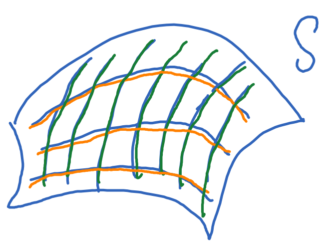
In general, if the surface is the graph of a function $z=f(x,y)$ then it has parametrization

$$\begin{cases} x = u \\ y = v \\ z = f(u,v) \end{cases}$$

Ex Find parametrization of the cone $z^2 = x^2+4y^2$, $z \geq 0$

$$\begin{cases} x = u \\ y = v \\ z = \sqrt{u^2+4v^2} \end{cases} \quad \text{or} \quad \begin{cases} x = r\cos\theta \\ y = \frac{1}{2} r\sin\theta \\ z = r \end{cases}$$

Tangent plane



$r(u,v) = (x(u,v), y(u,v), z(u,v))$

Green curve: fix u, move v.

Orange curve: fix v, move u.

r_u and r_v are two tangent vectors

Torus:

$$\begin{cases} x = (a + b\cos\phi)\cos\theta \\ y = (a + b\cos\phi)\sin\theta \\ z = a\sin\phi \end{cases}$$

Mobius strip:

$$\begin{cases} x = (a + s\cos bt)\cos t \\ y = (a + s\cos bt)\sin t \\ z = s\sin bt \end{cases}$$

$$r(u,v) = (1 + u - v,\ u + v^2,\ u^2 - v^2)$$

$$Q(2,3,3)$$